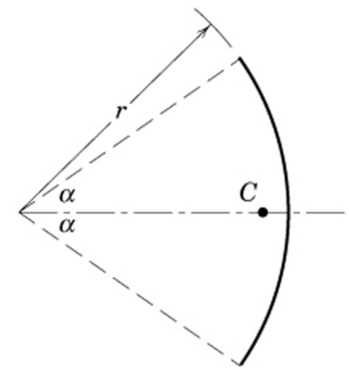
Center of Mass and Centroids

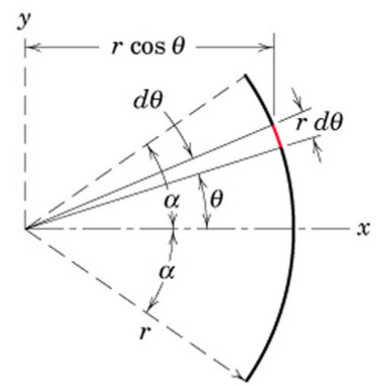
Examples: Centroids

Locate the centroid of the circular arc



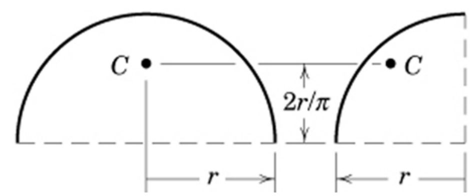
Solution: Polar coordinate system is better
Since the figure is symmetric: centroid lies on the x axis

Differential element of arc has length $dL = rd\Theta$
Total length of arc: $L = 2\alpha r$
x-coordinate of the centroid of differential element: $x = r\cos\Theta$



$$\bar{x} = \frac{\int x dL}{L} \quad \bar{y} = \frac{\int y dL}{L} \quad \bar{z} = \frac{\int z dL}{L}$$

$$L\bar{x} = \int x dL \qquad 2\alpha r\bar{x} = \int_{-\alpha}^{\alpha} (r cos\theta) r d\theta$$



$$2\alpha r\bar{x} = 2r^2 sin\alpha$$

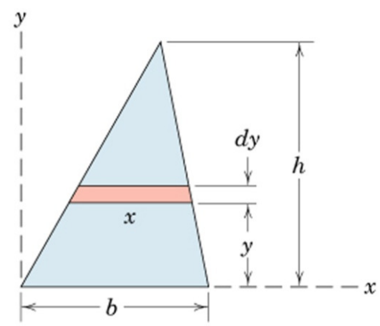
$$\bar{x} = \frac{r sin\alpha}{\alpha}$$

For a semi-circular arc: $2\alpha = \pi$ → centroid lies at $2r/\pi$

Center of Mass and Centroids

Examples: Centroids

Locate the centroid of the triangle along h from the base



Solution:

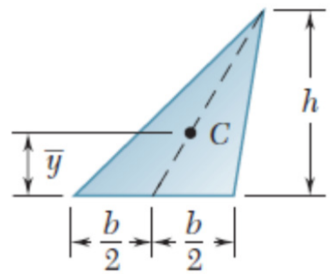
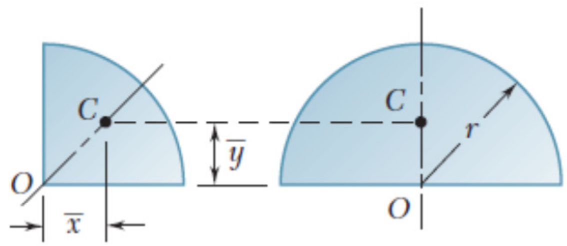
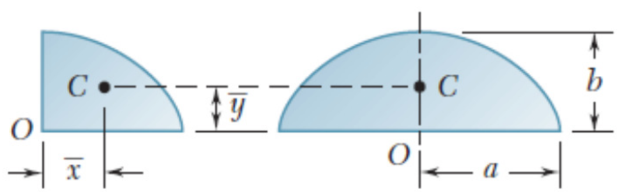
$$dA = xdy \qquad \frac{x}{(h-y)} = \frac{b}{h}$$

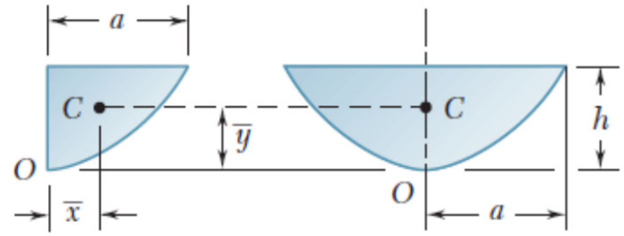
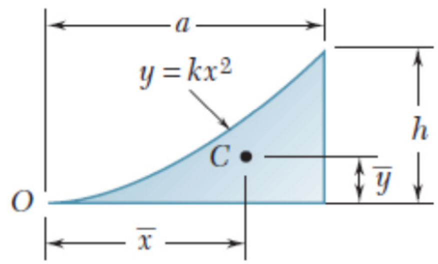
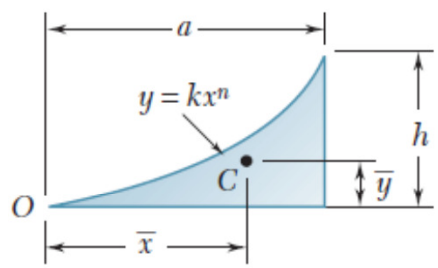
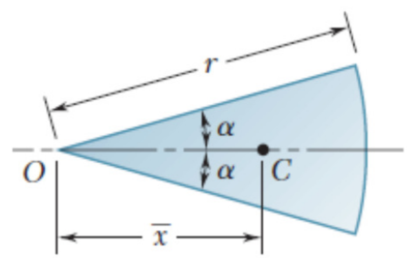
Total Area $A = \frac{1}{2}bh \qquad y = y_c$

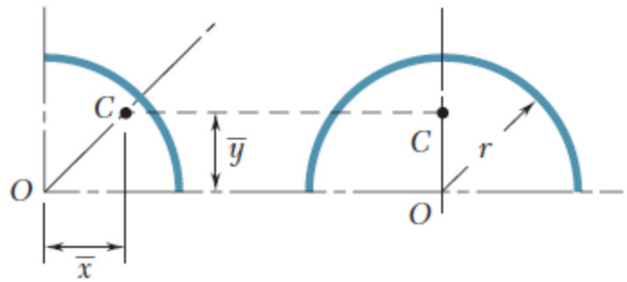
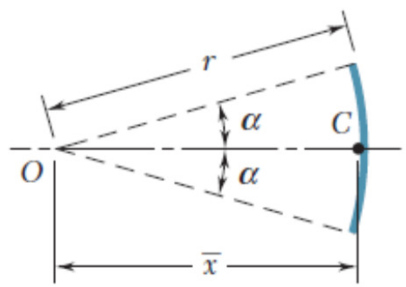
$$\bar{x} = \frac{\int x_c dA}{A} \quad \bar{y} = \frac{\int y_c dA}{A} \quad \bar{z} = \frac{\int z_c dA}{A}$$

$$A\bar{y} = \int y_c dA \quad \Rightarrow \frac{bh}{2}\bar{y} = \int_0^h y\frac{b(h-y)}{y}dy = \frac{bh^2}{6}$$

$$\boxed{\bar{y} = \frac{h}{3}}$$

Shape		$\overline{x}$	$\overline{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$

Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r \sin \alpha}{3\alpha}$	0	αr^2

Shape		$\bar{x}$	$\bar{y}$	Length
Quarter-circular arc		$\dfrac{2r}{\pi}$	$\dfrac{2r}{\pi}$	$\dfrac{\pi r}{2}$
Semicircular arc		0	$\dfrac{2r}{\pi}$	πr
Arc of circle		$\dfrac{r \sin \alpha}{\alpha}$	0	$2\alpha r$